

Convex optimisation for quantum theorists – Syllabus

Convex optimization has emerged as a critical tool for quantum information scientists, leveraging the natural convexity of quantum theory to offer powerful techniques for solving a variety of complex problems in the field. In this course we will develop these tools, with a focus on semidefinite optimization, analysing their mathematical properties and discussing their practical usage. The course will be driven by applications throughout quantum information science, using the tools we develop to solve important problems and provide new powerful perspectives on the field.

Lecture 1 (Convexity and convex optimisation)

Convex sets, cones and functions. Convex optimization problems and their properties.

Lecture 2 (Linear, semidefinite and conic optimisation)

Simple examples. Duality, Slater's theorem, Complementary slackness. Practical considerations and symmetries.

Lecture 3 (Basic applications to quantum theory)

Applications to: discrimination tasks, entanglement witnessing, cloning machines, complexity theory, channel coding.

Lecture 4 (The DPS hierarchy)

The separability problem and the DPS hierarchy. De Finetti theorem. Applications.

Lecture 5-6 (SDPs as proof tools)

Semidefinite representability of functions. Applications to proving properties of trace norm, fidelity and entropies. Additivity, data processing, inequalities.

Lecture 7 (Noncommutative polynomial optimization)

Basic problem, semidefinite hierarchy solution. Applications to Bell-nonlocality, cryptography, ground state energy problem.

Lecture 8 (Sum of squares)

Sum of Hermitian squares polynomials, properties, duality to NCPOPs, applications.